Engineering Notes

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Performance Estimation for Light Propeller Airplanes

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Nomenclature

$\mathcal{R}$ = $(\text{span})^2/S$ = aspect ratio of wing
C_D = D/qS
C_L = L/qS
D = total drag force
L = total lift force
mp = minimum power required for level flight at any altitude
P_m = $T_m V_m$, maximum power available at sea level
q = $\rho V^2/2 = \rho_0 \sigma V^2/2$
S = wing planform area
T = total thrust force
V = flight velocity
W = total weight of aircraft
w = maximum rate of climb with a fixed-pitch propeller
θ = climb angle
λ_p = $2W/\rho_0 S C_{D_e}$
λ_s = $2W\cos^2\theta/(\rho_0 S \pi \mathcal{R} e)$
ν^{-1} = $P_m/(2WV_m^3) = \tau/(3V_m^2)$
ρ = atmospheric mass density = $\sigma\rho_0$
σ = ρ/ρ_0
τ = $3P_m/(2WV_m)$
ϕ = altitude correction of static thrust

Subscripts

$(\ )_*$ = flight at maximum lift-to-drag ratio velocity $V_* = (\lambda_p \lambda_s)^{1/4}/\sqrt{\sigma}$
M = steady level flight at maximum velocity at any altitude
mp = flight at minimum power required at any altitude
w = flight at maximum rate of climb for fixed-pitch propeller

Introduction

THE nondimensional equations for the performance calculations of a jet airplane were simplified in Ref. 1 by using the varying velocity V_* and the constant minimum drag force D_* corresponding to the relations obtained at the maximum lift-to-drag ratio. The following analysis similarly simplifies the performance equations for a light propeller airplane by also using the velocity V_* after first introducing the constant density ratio ϕ_h to simplify the nondimensional equations. The constant density ratio ϕ_h corresponds to the theoretical absolute ceiling as given by Eq. 12 in Ref. 1. It is a constant that is only dependent on the airplane geometry, and the fixed-pitch propeller characteristics at sea level. These new equations reduce to those for the jet airplane in Ref. 1 when it is assumed that the thrust is independent of the level flight velocity at a constant altitude.

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It is also shown that the best blade angle for a fixed-pitch propeller is one that will produce the maximum available power at the maximum possible velocity for steady level flight at sea level.

Maximum Level Flight Velocity

The analysis in Ref. 1 was based on Oswald's nondimensional equation for the total airplane drag when the thrust vector is approximately parallel to the resultant velocity vector, namely,

$$D/W = \sigma V^2/\lambda_p + \lambda_s/(\sigma V^2) \tag{1}$$

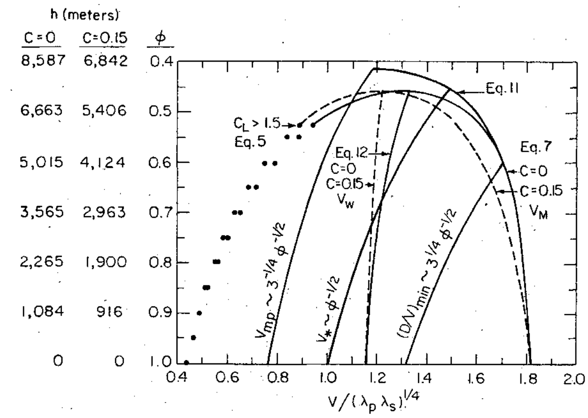

Fig. 1 Variation of nondimensional performance velocities with altitude for a typical light airplane defined by $C_{D_e} = 0.03$; $\pi\mathcal{R}e = 17$; $S = 13.94$ m^2; $W = 907.2$ kg; $\rho_0 = 1.225$ kg/m^3; $\lambda_p = 34{,}740$ (m/s)^2; $\lambda_s = 61.31$ (m/s)^2; $\nu = 63{,}680$ (m/s)^2; $\tau = 0.2287$, $\phi_h = 0.4585$; $P_m = 93.96$ kW (126 hp); $V_m = 69.54$ (m/s); $(\lambda_p\lambda_s)^{1/4} = 38.2$ (m/s) $= V_*(\sigma = 1)$.

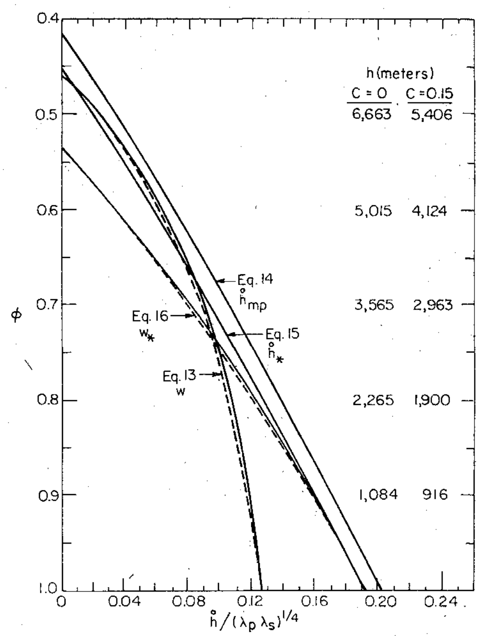

Fig. 2 Variation of nondimensional rate of climb with altitude for the light airplane defined in Fig. 1. The solid lines are for $C = 0$ and the dotted lines are for $C = 0.15$.

where the constants are given by[1]

$$\lambda_p = \frac{2W}{\rho_0 S C_{D_e}} \qquad \lambda_s = \frac{2W\cos^2\theta}{\rho_0 S \pi A\!\!R_e} \tag{2}$$

For light airplanes the climb angle θ is sufficiently small so that one can assume $\cos\theta \approx 1$ and λ_s reduces to the relation originally used by Oswald.[2]

The theoretical thrust available from a fixed-pitch propeller can be expressed as

$$T/W = \phi\tau - \sigma V^2 \nu^{-1} \tag{3}$$

where the constants are shown in Ref. 1 to be given by

$$\tau = \frac{3}{2}\frac{P_m}{WV_m} \qquad \nu^{-1} = \frac{1}{2}\frac{P_m}{WV_m^3} = \frac{\tau}{3V_m^2} \tag{4}$$

and the altitude correction is shown to be given by

$$\phi = \frac{\sigma - C}{1-C} \quad \text{for} \quad 0 \le C < 0.2 \quad \text{and} \quad \sigma = \frac{\rho_0}{\rho} \le 1$$

This altitude correction factor for the static thrust τW is often assumed to be approximated by $C=0$ or $\phi = \sigma$, but, as will be shown, this overestimates the performance at the higher altitudes for a propeller airplane. The term P_m represents the peak or maximum power available at the velocity V_m for a fixed-pitch propeller, as shown in Ref. 1. V_m can be increased by increasing the blade angle of the fixed-pitch propeller, as shown by Oswald.[2]

The maximum and minimum velocities for steady level flight are obtained when $T=D$ by equating Eqs. (1) and (3) so as to obtain

$$\sigma V^2 = \frac{\phi\tau}{2(\lambda_p^{-1} + \nu^{-1})}\left[1 \pm \sqrt{1 - \left(\frac{\phi_h}{\phi}\right)^2}\right] \tag{5}$$

The term

$$\phi_h = (2/\tau)\sqrt{\lambda_s(\lambda_p^{-1} + \nu^{-1})} \tag{6}$$

is a constant for any fixed-pitch propeller airplane, and it also happens to predict the density ratio at the theoretical absolute ceiling. This was first shown in Ref. 1 and is immediately evident in Eq. (5) because $\phi = \phi_h$ equates the maximum and minimum velocities, and as shown in Fig. 1, this can only occur at the absolute ceiling. Of course the minimum velocities calculated from Eq. (5) are valid only if the corresponding lift coefficients remain sufficiently small so that the limitations on the constants λ_p and λ_s are not exceeded, as described in Ref. 1. The dots in Fig. 1 represent $C_L > 1.5$, so level flight is obviously impossible at the minimum velocity defined by Eq. (5) unless the altitude is sufficiently high; for example, in Fig. 1 we find that Eq. (5) is acceptable for estimating the minimum velocities only if $\phi < 0.5$ for this light airplane, which is similar to a Cessna 120 or Piper PA-22. The data are taken from the drag polar given in Fig. 1 of Ref. 1, namely,

$$C_D = 0.03 + C_L^2/17$$

where $\pi A\!\!R e = 17$ is the corrected aspect ratio defined in Ref. 1.

The simplest nondimensional form for Eq. (5) is obtained by dividing it by $V_*^2 = \sqrt{\lambda_p\lambda_s}/\sigma$ (see Eq. 16 of Ref. 1) so that

$$\left(\frac{V_M}{V_*}\right)^2 = \left(\frac{\phi}{\phi_h}\right)(1+\lambda_p\nu^{-1})^{-1/2}\left[1+\sqrt{1-\left(\frac{\phi_h}{\phi}\right)^2}\right] = \frac{\sigma V_M^2}{\sqrt{\lambda_p\lambda_s}} \tag{7}$$

gives the maximum velocity (V_M) in steady level flight for a light airplane with a fixed-pitch propeller. This reduces to the corresponding relation for a light jet airplane (Eq. 18 of Ref. 1) if we let $\phi = \sigma$ for $C=0$, and assume the jet thrust only varies with altitude so that $T = \sigma T_0$, $\nu^{-1} = 0$, and the constant given by Eq. (6) reduces to

$$\sigma_h = \phi_h = (2/\tau)\sqrt{\lambda_s\lambda_p^{-1}} = (D_*/T_0) \tag{8}$$

This equation (Eq. 18 of Ref. 1) for the maximum steady level flight velocity for a light jet airplane can be further simplified by writing it as

$$\frac{V_M^2}{\sqrt{\lambda_p\lambda_s}} = \frac{T_0}{D_*}\left[1+\sqrt{1-\left(\frac{D_*}{\sigma T_0}\right)^2}\right] \tag{9}$$

where $\sqrt{\lambda_p\lambda_s} = \sigma V_*^2$ is a constant that is the square of the velocity for the maximum lift-to-drag ratio at sea level ($\sigma = 1$).

Also shown in Fig. 1 is the maximum possible level flight velocity that is obtained by assuming that the maximum available power is independent of speed and $C=0$ so that as an upper limit for a variable-pitch propeller,

$$TV_m = \sigma P_m = DV_m = \sigma V_m^3 W/\lambda_p + W\lambda_s/(\sigma V_m) \tag{10}$$

where $P_m = 94$ kW (126 hp) is the maximum available power at sea level. This biquadratic equation is easily solved by iteration in the following form:

$$V_m \rightarrow V_n = \left(\frac{P_m\lambda_p}{W} - \frac{\lambda_p\lambda_s}{\sigma^2 V_{n-1}}\right)^{1/3} \tag{11}$$

where the first iteration is given by $V_m \approx (P_m\lambda p/W)^{1/3} = 71.59$ m/s (234.88 ft/s). The variation of V_m with $\phi = \sigma$ as shown in Fig. 1 is practically the same as Eq. (7) for the Oswald fixed-pitch propeller for $\phi > 0.5$, only the absolute ceiling calculations differ appreciably. This shows that Oswald's[2] recommendation that the blade angle be selected so as to absorb the maximum available power at the sea level maximum velocity does provide excellent performance. It should be noted that the greater velocities predicted by Eq. (11) at the higher altitudes cannot be attained since C_L is too large to justify the assumption of constant values for λ_p and λ_s. For example, Eq. (11) at absolute ceiling corresponds to $\sigma_h = 0.4143$ (for $C=0$ at 8292 m) and $C_{L_h} = 1.24$, whereas Eq. (7) corresponds to $C_{L_h} = 0.888$ at either $\sigma_h = 0.4585$ (for $C=0$ at 7423 m), or at $\sigma_h = 0.5397$ (for $C=0.15$ at 5981 m). Obviously the latter absolute ceiling of 5981 m (19,623 ft) is a better approximation.

Maximum Rate of Climb

If we again divide by $V_*^2 = \sqrt{\lambda_p\lambda_s}\,\nabla\sigma$ and introduce Eq. (6) for ϕ_h into Eq. 14 of Ref. 1, we obtain

$$\left(\frac{V_w}{V_*}\right)^2 = \frac{1}{3}\left(\frac{\phi}{\phi_h}\right)(1+\lambda_p\nu^{-1})^{-1/2} \times \left[1+\sqrt{1+3\left(\frac{\phi_h}{\phi}\right)^2}\right] = \frac{\sigma V_w^2}{\sqrt{\lambda_p\lambda_s}} \tag{12}$$

for the climbing flight velocity (V_w) that produces the maximum rate of climb (w), which is similarly given by Eq. 15 of Ref. 1 as $\dot{h}_{max} = w = V_w \sin\theta_w$,

$$\sin\theta_w = \frac{2}{3}(\phi\tau)\left[2-\sqrt{1+3\left(\frac{\phi_h}{\phi}\right)^2}\right] = \frac{w}{V_w} \tag{13}$$

The values calculated from Eq. (12) for $V_w/(\lambda_p\lambda_s)^{1/4}$ are shown in Fig. 1, while those from Eq. (13) for $w/(\lambda_p\lambda_s)^{1/4}$ are shown in Fig. 2 for the case of a fixed-pitch propeller that satisfies Oswald's[2] suggestion that the best performance is realized by selecting the propeller blade angle that absorbs the maximum available power (94 kW = 126 hp) at the maximum possible level flight velocity (69.54 m/s = 228.15 ft/s) at sea level ($\sigma = 1 = \phi$). The velocity variations are shown for $C = 0$ and 0.15, and it is seen that these correspond to absolute ceilings of 7423 m (23,354 ft) and 5981 m (19,623 ft), respectively. A comparison with the flight data for this type of light airplane equipped with a variable-pitch (constant rpm) propeller indicates the absolute ceiling would be approximately 5500 m (18,000 ft).

In Fig. 2 we also show the theoretical upper limit for the rate of climb ($\dot{h}_{mp}$) based on the so-called "ideal airplane" whose available power is independent of speed so that $\dot{h}_{mp}$ is produced at V_{mp}, the velocity for minimum required power, as follows:

$$\dot{h}_{mp} = (\sigma P_m - D_{mp} V_{mp})/W = \frac{\sigma P_m}{W} - \frac{4}{3^{3/4}} \lambda_s^{3/4} \lambda_p^{-1/4} / \sqrt{\sigma}$$

$$V_{mp} = V_*/3^{1/4} \qquad C_{L_{mp}} = \sqrt{3} C_{L*} = 1.24 \tag{14}$$

(e.g., see Eq. 28 of Ref. 1). However, as first pointed out by Oswald,[2] the actual minimum power nearly always occurs at a flight velocity considerably greater than V_{mp}, but less than V_*. Therefore we also show the rate of climb ($\dot{h}_*$) at the climbing velocity V_* with the same assumption of power available being independent of speed, so that for constant $C_{L*} = 0.714$,

$$\dot{h}_* = (\sigma P_m / W) - 2\lambda_s^{3/4} \lambda_p^{-1/4} / \sqrt{\sigma} \tag{15}$$

Figure 2 clearly indicates that Eq. (15) for $\dot{h}_* = 0$ with $C = 0.15$ gives a much better approximation to the actual absolute ceiling at $\sigma_h = 0.5342$ corresponding to 6073 m (19,923 ft), than does the value from Eq. (14) for $\dot{h}_{mp} = 0$ and $C = 0.15$, which is $\sigma_h = 0.5021$, corresponding to 6626 m (21,738 ft). Figure 2 clearly shows that the assumption of $\sigma = \phi$ ($C = 0$) results in a much too high estimate of the ceiling. The reason why Eq. (14) usually overestimates the rate of climb is that $C_{L_{mp}}$ is too large to justify the use of the constant values of λ_p and λ_s, as seen in Fig. 1 of Ref. 1. On the other hand Eq. (15) corresponds to $C_{L*} < 1$ so its predictions are more valid, although as seen in Fig. 2, $\dot{h}_*$ can be exceeded by the rate of climb (w) from Eq. (12) for a fixed-pitch propeller because at the higher altitudes $C_{L_w} > C_{L*}$. However, the predictions from Eq. (12) are valid for the example used in Figs. 1 and 2, since the C_L involved are all less than $C_{L_h} = 0.888$, and therefore the use of the constant values of λ_p and λ_s are justified. It should be noted that Fig. 2 confirms Oswald's[2] opinion that the best performance of a fixed-pitch propeller occurs when the propeller blade angle is selected to absorb the engine's sea level maximum P_m at the maximum possible steady level flight velocity. Oswald's propeller selection performs excellently as the altitude increases, nearly matching the ideal airplane with constant power assumption while still maintaining the basis of Eq. (1) with constant values for λ_p and λ_s, since $C_{L_w} \leq 0.888$.

The important restriction on Eqs. (12) and (13) is that C_{L_w} must not greatly exceed unity in order to justify the application of Eqs. (1) and (2). For example, if the propeller blade angle is decreased so that $V_m = (\lambda_p \lambda_s)^{1/4} = 38.2$ m/s (125.3 ft/s) in order to increase the rate of climb at sea level by 54%, then as the altitude increases, Eq. (12) predicts such low values for V_w that $C_{L_w} > 1$ for $\phi < 0.8$, the maximum value of $C_{L_w} = 1.48$ occurs at $\phi_h = 0.42$, and because the drag predicted by Eq. (1) is exceeded, Eq. (13) overestimates the rate of climb. In this case it is better to assume that the climb occurs at the constant values of $C_{L*} = 0.7141$ and $D_* = 747.5$ N (168 lb) at the climbing velocity given by $V_* = (\lambda_p \lambda_s)^{1/4} / \sqrt{\sigma}$, so the rate of climb is given by

$$w_* = \left(\frac{T_* - D_*}{W}\right) V_* = \left[\tau_* \left(\phi - \frac{1}{3}\right) - \frac{D_*}{W}\right] \frac{(\lambda_p \lambda_s)^{1/4}}{\sqrt{\sigma}} \tag{16}$$

where

$$\tau_* = \frac{3}{2} \frac{P_m / W}{(\lambda_p \lambda_s)^{1/4}} \qquad \nu_*^{-1} = \frac{\tau_*}{3 (\lambda_p \lambda_s)^{1/2}}$$

Figure 2 indicates that Eqs. (15) and (16) may provide a satisfactory upper and lower limit for the rate of climb for any variable pitch propeller. Equation (16) yields $\sigma_h = 0.5359$, giving an absolute ceiling of 6044 m (19,829 ft) for $C = 0$, and 4929 m (16,174 ft) for $C = 0.15$. The latter is too low, so a calculation with $C = 0.0732$ gives an absolute ceiling of 5500 m (18,000 ft), which is reasonable. This shows how important the determination of $\phi(\sigma)$ is for performance prediction as the altitude increases. Perhaps a review of existing flight data could produce a better relation for ϕ/σ than the empirical relation used in Eq. (3) and in Refs. 1 and 2.

Finally, Fig. 1 shows the velocity for the minimum value of (D/V), as given by $V/(\lambda_p \lambda_s)^{1/4} = 3^{1/4} \sigma^{-1/2}$ for $\sigma = \phi (C = 0)$. It is seen that at this "optimum cruise speed,"[3] the Oswald fixed-pitch propeller Eq. (7), is as effective as Eq. (11), the upper limit for a variable-pitch propeller whose power output does not decrease with speed.

References

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Transonic Shock/Turbulent Boundary-Layer Interaction on Curved Surfaces

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Introduction

An understanding of transonic shock/boundary-layer interaction (SBLI) is important in the aerodynamic design of high-speed aircraft wings (both ordinary and circulation controlled), turbine and cascade blades in turbomachinery, and air-breathing engine inlets and diffusors. Since these applications often involve curved surfaces, and since a singularity is associated with a normal shock on a curved surface in purely inviscid flow,[1] the influence of wall curvature on transonic shock/boundary-layer interaction is an important basic and practical question. This problem is addressed here for the case of nonseparating high Reynolds number ($10^5 < Re_L \leq 10^8$) turbulent boundary layers on convex surfaces with a curvature boundary-layer thickness product $K\delta_0$ in the practical range of $0 \leq K\delta_0 \leq 0.02$.

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